NOTRE DAME UNIVERSITY

Department of Mathematics and Statistics

MAT 326

PROBABILITY AND STATISTICS FOR ENGINEERS

DURATION 55 MINUTES

NAME: . ...

SECTION: ....

INSTRUCTOR: ....

GRADE:

1	2	MC	Total
10	07	8×5 =40	$\frac{57}{100}$ +15 = $\frac{72}{100}$

THE DEBATE CLUB

1) (20 points) The length of time " in years " to complete a certain construction project is a continuous random variable X whose probability density function is given by

$$f(x) = \begin{cases} \frac{1}{8}(x-1) & for\ 1 \le x \le 5 \\ 0 & elsewhere \end{cases}$$

a) Find the average time to complete this construction project. (5 points)

$E(Y) = \frac{1}{8}\int_1^5 (x-1)\,dx = \frac{1}{8}\left[\frac{x^2}{2} - x\right]_1^5 = \frac{1}{8}\left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right] = \frac{1}{8}(8) = 1$

$E(X) = \frac{1}{8}\int_1^5 (x^2 - x)\,dx = \frac{1}{8}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^5 = \frac{1}{8}\left[\left(\frac{125}{3} - \frac{25}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$

$= \frac{1}{8}\left[\frac{175}{6} + \frac{1}{6}\right] = \frac{11}{3}$

$\boxed{Average = \frac{11}{3}}$

$\boxed{E(X) = \frac{11}{3}}$

b) The profit Y " in $1000 " after completing this project is given by Y = 650 – 100 X. Find the expected value and the standard deviation of the profit knowing that $\sigma_X = 0.941$ (5 Points)

$E(Y) = 650 - 100E(X) = 650 - 100 \times \frac{11}{3} = \frac{850}{3}$

$\sigma_Y^2 = \cancel{650^2 - 100^2 \sigma_X^2} = 422500 - 10000 \times 0.885481$

$\sigma_Y^2 = 413645.19$

$\sigma_Y = 643.15$

(−3)

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c) Find the cumulative distribution function F(x) of X. (6 points)

$F(x) = \frac{1}{4}\int_{1}^{5} (x-1)\,dx = \frac{1}{4}\left[\frac{x^2}{2} - x\right]_1^5 = 1$

(−6)

d) Use F(x) to find the probability of making a profit of at least 350000 dollars. (4 points)

~~$P(Y \geq 350000) = 1 - F(35000) = 1 - \left\{\frac{1}{4}\left(\frac{35000^2}{2} - 35000\right)\right\}$~~

~~$P(Y \geq 650 - 100x)$~~

$P(Y \geq 350000) = P(650 - 100x \geq 350000)$

$P(-100x \geq [illegible])$

$P(x \leq [illegible])$

$3453.5 = F(3453.5)$

(−1)

$F(3453.5) = \frac{1}{4}\left[\frac{x^2}{2} - x\right] = 1.673022 1$

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$P(\bar{C}/S) = 1 - P($

2) (10 points) A construction job may not be completed on time. The probability that the job will be completed on time is 0.92 when there is no strike, and it is 0.65 when there is a strike. The probability that there will be a strike is only 0.25

a) What is the probability that the job will not be completed on time? (5 points)

$P = 0.296$

S: {There is a strike}

$\bar{S}$ {There is not a strike}

C: {job complete on time}

$\bar{C}$: {job not complete on time}

$P(\bar{C}) = P(\bar{C} \cap S) + P(C \cap \bar{S})$

$= P(C/S) \cdot P(S) + P(C/\bar{S}) \times P(\bar{S})$

$= 0.65 \times 0.25 + 0.92 \times 0.75$

$P(C) = 0.8525$

$P(\bar{C}) = 1 - P(C) = 1 - 0.8525 = 0.1475$

$P($

+5

b) If the job was not completed on time, find the probability that there was no strike (5 points)

$P(\bar{S}/\bar{C}) = \frac{P(\bar{S} \cap \bar{C})}{P(\bar{C})} =$

$P(S \cap \bar{C})$

$P(S/\bar{C}) \times P(\bar{C})$

$P(\bar{C}/\bar{S}) = 1 - P(C/\bar{S})$

$P(C/\bar{S}) = \frac{P(\bar{S} \cap \bar{C})}{P(\bar{S})} = 0.92 \times 0.75$

$P(\bar{S}/\bar{C}) = \frac{P(\bar{S} \cap \bar{C})}{P(\bar{C})}$

+2

$\frac{1 - (0.92 \times 0.75)}{0.1475} =$

THE DEBATE CLUB

For each of the following questions, circle the right answer. There is only <u>one</u> correct answer for each question. If you circle more than one answer per question, your answer would be considered incorrect

*** Suppose you are throwing a fair die. The probability of getting an even number for the second time on the 4-th trial is

a) $\frac{3}{16}$ b) $\frac{1}{16}$ c) $\frac{2}{16}$ d) None of these

$\frac{3}{6}$

*** Let X be a geometric random variable. If P (X ≥ 2) = 0.4, then P(X= 2) equals to

a) 0.4 b) 0.6 c) 0.24 d) None of these

$\sum_{2}^{\infty} p^{2} \times (1-p)$

$P(X \geq 2) = 0.4$

$1 - P(X < 2) = P(X=1) + P(X=0)$

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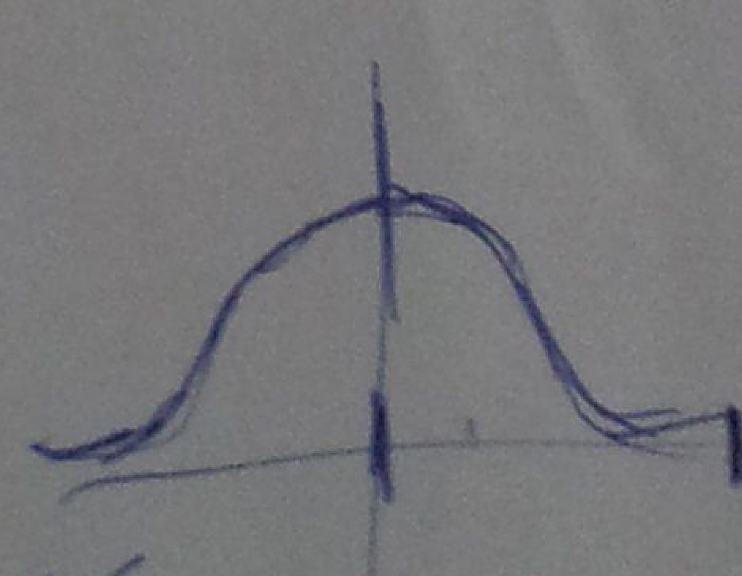
*** Let X be a positive random variable with mean = 6, and variance = 16. The lower bound of $P(0 \le X \le 14)$ is

a) 0.556 b) 0.9104 c) 0.75 d) None of these

X $P(Z < \frac{14-6}{16}) = P(Z < \frac{8}{16}) = P(Z < 0.5) = 0.5 - 0.1915$

$P(Z < \frac{0-6}{16}) = Z < \frac{6}{16}$



$P(Z < -0.375) = 0.5 + 1.15$

*** If two events A and B are mutually exclusive with P(A) = 0.2 and P(B) = 0.3 , then we can say that A and B are

$P(A.B)$

a) Independent with P (A.B) = 0.06
b) Dependent with p(A.B) = 0
c) Independent with P(A.B)=0
d) We need more information to make a decision

$P(A \cap B) =$

X

***If A and B are two events , with P(A) = 0.4, P(B)= 0.6, and P(A.B)= 0.06, then $P(\bar{A} \cap \bar{B})$ equals to

a) 0.06　　b) 0.94　　c) 0.24　　d) None of these

$\cap$

$1-(P(A)+P(B)-0.06)$

$P(A)=0.4 \quad P(B)=0.6$
$P(A \cap B)=0.06$
$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$
$= 1 - P(A \cup B)$

*** If X is a poisson random variable with a variance of 4, then P(X=1) is

a) $8e^{-4}$　　b) $2e^{-2}$　　c) $2e^{-4}$　　d) None of these

$\frac{e^{-4} \times \lambda^{-1}}{1} = e^{-4} \times 4 = e^{-4} \times \frac{1}{4}$ $\quad 4e^{-4}$ $\quad \frac{e^{-\lambda} \times \lambda^{-x}}{x!} = \frac{e^{-4} \times 4^{-1}}{1}$

***The number of customers arrive at a certain small bank follows a poisson distribution with λ= 15/hour. At 9:00 Am, one customer arrives, what is the probability that the next customer will arrive after 9:10 Am?

a) $e^{-2.5}$　　b) $e^{-1.5}$　　c) $1- e^{-2.5}$　　d) None of these

$e^{-\frac{1}{15}/\frac{1}{6}} = e^{-\frac{1}{15} \times 6}$

$e^{-\frac{1}{1.5}/\frac{1}{6}} = e^{\frac{1}{1.5} \times 6}$

$1.5/\frac{1}{6}$

$\lambda = 15 h$

$\frac{15}{60}$

$\frac{1}{4}$

$\frac{15}{60}$

$15 \to 60$
$\lambda \quad 10$

$\frac{10}{60} = \frac{1}{6}$

THE DEBATE CLUB

$P(X>3)$

*** X is an exponential random variable with the mean β. If $P(X > 3) = 0.2231$, then $P(X > 6)$ equals to

a) 0.4462 b) 0.5538 c) 0.0498 d) None of these

$\left[-e^{-\frac{x}{\sigma}}\right]_3^{\infty}$ $-(-e^{\frac{3}{\sigma}})$ $e^{\frac{3}{\sigma}} = 0.2231$

$e^{\frac{6}{-2}} =$ $\frac{3}{\sigma} = -1.5$

$\sigma = -2$

*** If X is a Binomial random variable with mean = 2, and variance of 1.2, then $P(X=0)$ is equal to

a) 0.0102 b) 0.0778 c) 0.92224 d) None of these

$C_0^7 \times (0.28)^0 \times (0.72)^7$ $np = 2$ $\sigma = np$ $np(1-p) = 1.44$

$np = 2$ $n = \frac{2}{p}$ $n = 7$

$np(1-p) = 1.44$

$np - np^2 = 1.44$

$2(1-p) = 1.44$ $(\frac{2}{p} \times p) - \frac{2}{p} \times p^2 = 1.44$

$1 - p = 0.6$ $2 - 2p = 1.44$ $p = 0.4$

$1 \times 1 \times (0.6)^5$

*** In a certain mall it is known that 60% of the shoppers pay their bills by cash money. Three shoppers are paying their bills, What is the probability that the first one will pay cash money?

a) 0.288 b) 0.936 c) 0.60 d) None of these

$p = 0.6$ cash

$P(C/b) = \frac{P(C \cap b)}{P(b)} = \frac{P(C \cap b)}{P(b/C) \times P(C)}$

*** X is a normal random variable with $\mu = 70$, and $\sigma^2 = 100$. If $P(X > x) = 0.8413$, then x equals to

a) 75 b) 60 c) 80 d) None of these

$P(Z > \frac{x-70}{10}) = 0.8413$

$\frac{x-70}{10} = 1$

$x - 70 = 10$

$x = 60$

THE DEBATE CLUB

*** The probability distribution of a discrete random variable X is given by

$P(X=x) = \frac{|x-1|}{15}$ for $x = -2, -1, 0, 1, 2$.

E(X) equals to

a) 0.0 b) $-\frac{16}{15}$ c) $\frac{16}{15}$ d) None of these

$P(x=-2) = \frac{1}{3}$ $E(x=-2) = \frac{1}{15}$ $E(x=1) = \frac{2}{15}$

$P(x=-1) = \frac{1}{15}$ $E(x=-1) = -\frac{2}{3}$ $E(x=2) = \frac{2}{15}$

$P(x=0) = \frac{1}{5}$ $E(x=-1) = -\frac{1}{15}$ $E(x) =$

$P(x=1) = \frac{2}{15}$ $E(x=0) = 0$

*** If the cumulative distribution function F(x) of a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & \text{for } x \le 1 \\ \frac{1}{44}(2x^2 - x - 1) & \text{for } 1 \le x \le 5 \\ 1 & \text{for } x > 5 \end{cases}$$

then P(X> 2) equals to

a) $\frac{87}{88}$ b) $\frac{78}{88}$ c) $\frac{15}{88}$ d) None of these

$F(x) = \frac{1}{44} \int_2^5 (2x^2 - x - 1)\,dx$ $\frac{1}{44}\left[2x^2 - x - 1\right]_2^5$ $\frac{1}{44}\left[(\ldots - 5 - 1) - (\ldots\right.$

*** If X is a discrete random variable with P(X = x) = Kx for x= 1,2,3,4. Then the value of K is

a) 10 b) $\frac{1}{10}$ c) $\frac{1}{4}$ d) None of these

$P(X=1) = k$ $10k$

$P(X=2) = 2k$

$P(X=3) = 3k$

$P(X=4) = 4k$

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